

The Ratio of CO to Total Gas Mass in High Redshift Galaxies

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ABSTRACT

Walter et al. (2012) have recently identified the J=6-5, 5-4, and 2-1 CO rotational emission lines, and [CII] fine-structure emission line from the star-forming interstellar medium in the high-redshift submillimeter source HDF 850.1, at $z = 5.183$. We employ large velocity gradient (LVG) modeling to analyze the spectra of this source assuming the [CII] and CO emissions originate from (i) separate unvirialized regions, (ii) separate virialized regions, (iii) uniformly mixed unvirialized region, and (iv) uniformly mixed virialized regions. We present the best fit set of parameters, including for each case the ratio α between the total hydrogen/helium gas mass and the CO(1-0) line luminosity. We also present computations of the ratio of H₂ mass to [CII] line-luminosity for optically thin conditions, for a range of gas temperatures and densities, for direct conversion of [CII] line-luminosities to “dark-H₂” masses. For HDF 850.1 we find that a model in which the CO and C⁺ are uniformly mixed in gas that is shielded from UV radiation, requires a cosmic-ray or X-ray ionization rate of $\zeta \approx 10^{-13} \text{ s}^{-1}$, plausibly consistent with the large star-formation rate ($\sim 10^3 \text{ M}_\odot \text{ yr}^{-1}$) observed in this source. Enforcing the cosmological constraint posed by the abundance of dark matter halos in the standard Λ CDM cosmology and taking into account other possible contributions to the total gas mass, we find that three of these four models are less likely at the 2σ level. We conclude that modeling HDF 850.1’s ISM as a collection of unvirialized molecular clouds with distinct CO and C⁺ layers, for which $\alpha = 0.6 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$ for the CO to H₂ mass-to-luminosity ratio, (similar to the standard ULIRG value), is most consistent with the Λ CDM cosmology.

Key words: cosmology: theory – galaxies: high-redshift – galaxies: ISM – galaxies: mass function.

1 INTRODUCTION

Observations of high-redshift CO spectral line emissions have greatly increased our knowledge of galaxy assembly in the early Universe. At redshifts $z \sim 2$, this includes the discovery of turbulent star-forming disks with cold-gas mass fractions and star-formation rates significantly larger than in present day galaxies (Daddi et al. 2010; Genzel et al. 2012; Magnelli et al. 2012; Tacconi et al. 2010, 2012). The high star-formation rates are correlated with large gas masses and luminous CO emission lines (Kennicutt & Evans 2012) observable to very high redshifts. A prominent example is the luminous submillimeter and Hubble-Deep-Field source HDF 850.1, which is at a redshift of $z = 5.183$ as determined

by recent detections of CO(6-5), CO(5-4), and CO(2-1) rotational line emissions, and also [CII] fine-structure emission in this source (Walter et al. 2012). The high redshift of HDF 850.1 offers the opportunity of setting cosmological constraints on the conversion factor from CO line luminosities to gas masses, via the implied dark matter masses and the expected cosmic volume density of halos of a given mass. Such analysis is the subject of our paper.

CO emitting molecular clouds, which provide the raw material for star formation, are usually assumed to have undergone complete conversion from atomic to molecular hydrogen. However, since H₂ has strongly forbidden rotational transitions and requires high temperatures ($\sim 500 \text{ K}$) to excite its rotational lines, it is a poor tracer of cold ($\lesssim 100 \text{ K}$) molecular gas. Determining H₂ gas masses in the interstellar medium (ISM) of galaxies has therefore relied on tracer molecules. In particular, ¹²CO is the most commonly employed tracer of ISM clouds; aside from being the

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most abundant molecule after H_2 , CO has a weak dipole moment ($\mu_e = 0.11$ Debye) and its rotational levels are thus excited and thermalized by collisions with H_2 at relatively low molecular hydrogen densities (Solomon & Vanden Bout 2005).

The molecular hydrogen gas mass is often obtained from the CO luminosity by adopting a mass-to-luminosity conversion factor $\alpha = M_{\text{H}_2}/L'_{\text{CO}(1-0)}$ between the H_2 mass and the $J=1-0$ 115 GHz CO rotational transition (Bolatto et al. 2013). The value for α has been empirically calibrated for the Milky Way Galaxy by three independent techniques: (i) correlation of optical extinction with CO column densities in interstellar dark clouds (Dickman 1978); (ii) correlation of gamma-ray flux with the CO line flux in the Galactic molecular ring (Bloemen et al. 1986; Strong et al. 1988); and (iii) observed relations between the virial mass and CO line luminosity for Galactic GMCs (Solomon et al. 1987). These methods have all arrived at the conclusion that the conversion factor in our Galaxy is fairly constant. The standard Galactic value is $\alpha = 4.6 \text{ M}_\odot/(\text{K km}^{-1} \text{ pc}^2)$. Subsequent studies of CO emission from unvirialized regions in Ultra-Luminous Infrared Galaxies (ULIRGs) found a significantly smaller ratio. For such systems, $\alpha = 0.8 \text{ M}_\odot/(\text{K km}^{-1} \text{ pc}^2)$ (Downes & Solomon 1998). These values have been adopted by many (Sanders et al. 1988; Tinney et al. 1990; Wang et al. 1991; Walter et al. 2012) to convert CO $J=1-0$ line observations to total molecular gas masses, but without consideration of the dependence of α on the average molecular gas conditions found in the sources being considered. Since all current observational studies of α leave its range and dependence on the average density, temperature, and kinetic state of the molecular gas still largely unexplored, its applicability to other systems in the local or distant Universe is uncertain (Papadopoulos et al. 2012).

In this paper, we estimate the CO emitting gas masses in HDF 850.1 using the large-velocity-gradient (LVG) formalism to fit the observed emission line spectral energy distribution (SED) for a variety of model configurations, and for a comparison to the Galactic and ULIRG conversions. In §2 we outline the details of the LVG approach, including an overview of the escape probability method and a derivation of the gas mass from the line intensity of the modeled source. In §3, we present the best fit set of parameters that reproduce HDF 850.1's detected lines and calculate the corresponding molecular gas mass, assuming the CO and [CII] emission lines originate from (i) separate unvirialized regions, (ii) separate virialized regions, (iii) uniformly mixed unvirialized regions, and (iv) uniformly mixed virialized regions. The inferred gas masses enable us to set lower-limits on the dark-matter halo mass for HDF 850.1. In §4 we compare the estimated halo-masses to the number of such objects expected at high redshift in Λ CDM cosmology, and show that models with lower values of α are favored. We conclude with a discussion of our findings and their implications in §5.

2 LARGE VELOCITY GRADIENT MODEL

We start by describing our procedure for quantitatively analyzing the [CII] and CO emission lines detected at the position of HDF 850.1 using the large velocity gradient (LVG)

approximation. We consider a multi-level system with population densities of the i th level given by n_i . The equations of statistical equilibrium can then be written as :

$$n_i \sum_{j \neq i}^l R_{ij} = \sum_{j \neq i}^l n_j R_{ji} \quad (1)$$

where l is the total number of levels included; since the set of l statistical equations is not independent, one equation may be replaced by the conservation equation

$$n_{\text{tot}} = \sum_{j=0}^l n_j \quad (2)$$

where n_{tot} is the number density of the given species in all levels. In our application, $n_{\text{tot}} = n_{\text{CO}}$. Following the notation of Poelman & Spaans (2005), R_{ij} is given in terms of the Einstein coefficients, A_{ij} and B_{ij} , and the collisional excitation ($i < j$) and de-excitation rates ($i > j$) C_{ij} :

$$R_{ij} = \begin{cases} A_{ij} + B_{ij} \langle J_{ij} \rangle + C_{ij}, & (i > j) \\ B_{ij} \langle J_{ij} \rangle + C_{ij}, & (i < j) \end{cases} \quad (3)$$

where $\langle J_{ij} \rangle$ is the mean intensity corresponding to the transition from level i to j averaged over the local line profile function $\phi(\nu_{ij})$. The total collisional rates C_{ij} depend on the individual temperature-dependent rate coefficients and collision partners, usually H_2 and H for CO

The difficulty in solving this problem is that the mean intensity at any location in the source is a function of the emission and varying excitation state of the gas all over the rest of the source, and is thus a nonlocal quantity. To obtain a general solution of the coupled sets of equations describing radiative transfer and statistical equilibrium, we adopt the approach developed by Sobolev (1960) and extended by Castor (1970) and Lucy (1971) and assume the existence of a large velocity gradient in dense clouds. This assumption is justified given the interstellar molecular line widths which range from a few up to a few tens of kilometers per second, far in excess of plausible thermal velocities in the clouds (Goldreich & Kwan 1974). They suggest that these observed velocity differences arise from large-scale, systematic, velocity gradients across the cloud, a hypothesis that lies in accord with the constraints provided by observation and theory.

In the limit that the thermal velocity in the cloud is much smaller than the velocity gradient, the value of $\langle J_{ij} \rangle$ at any point in the cloud, when integrated over the line profile, depends only upon the local value of the source function and upon the probability that a photon emitted at that point will escape from the cloud without further interaction. Thus $\langle J_{ij} \rangle$ becomes a purely local quantity, given by:

$$\langle J_{ij} \rangle = (1 - \beta_{ij}) S_{ij} + \beta_{ij} B(\nu_{ij}, T_B) \quad (4)$$

where S_{ij} is the line source function,

$$S_{ij} = \frac{2h\nu_{ij}^3}{c^2} \left(\frac{g_i n_j}{g_j n_i} - 1 \right)^{-1} \quad (5)$$

assumed constant through the medium. In this expression g_i and g_j are the statistical weights of levels i and j respectively, β_{ij} is the ‘‘photon escape probability’’, and $B(\nu_{ij}, T_B)$ is the background radiation with temperature T_B . In our models we set T_B to the CMB temperature of

Table 1. CO and [CII] Line Observations of HDF 850.1

Line	ν_{obs} [GHz]	Integrated Flux Density [Jy km s ⁻¹]	Luminosity [10 ¹⁰ K km s ⁻¹ pc ²]	Intensity [erg(s cm ² str) ⁻¹]
CO(2-1)	37.286	0.17±0.04	4.1±0.9	(2.16±0.51)×10 ⁻⁹
CO(5-4)	93.202	0.5±0.1	1.9±0.4	(1.59±0.32)×10 ⁻⁸
CO(6-5)	111.835	0.39±0.1	1.0±0.3	(1.49±0.82)×10 ⁻⁸
[CII]	307.383	14.6±0.3	5.0±0.1	(1.5±0.03)×10 ⁻⁶

Detection of four lines tracing the star-forming interstellar medium in HDF 850.1 (Walter et al. 2012)

16.9 K at $z = 5.183$, and we ignore contributions from warm dust (da Cunha et al. 2013).

For a spherical homogenous collapsing cloud, the probability that a photon emitted in the transition from level i to level j escapes the cloud is given by

$$\beta_{ij} = \frac{1 - e^{-\tau_{ij}}}{\tau_{ij}} \quad (6)$$

where τ_{ij} is the optical depth in the line,

$$\tau_{ij} = \frac{A_{ij}}{8\pi} \frac{c^3}{\nu_{ij}^3} \frac{n_{tot}}{dv/dr} \frac{n_j g_i}{n_{tot} g_j} \left(1 - \frac{g_j n_i}{g_i n_j}\right). \quad (7)$$

The equations of statistical equilibrium are therefore reduced to the simplified form

$$\sum_{j \neq i}^l (n_i C_{ij} - n_j C_{ji}) + n_i \sum_{\substack{j \neq i \\ j < i}}^l A_{ij} \beta_{ij} - \sum_{\substack{j \neq i \\ j > i}}^l n_j A_{ji} \beta_{ji} = 0 \quad (8)$$

and can be solved through an iterative process to give the fractional level populations n_i/n_{tot} (for a given choice of densities for the collision partners, usually H₂ but also H, and kinetic temperature T_{kin}). Assuming the telescope beam contains a large number of these identical homogeneous collapsing clouds (e.g., Hailey-Dunsheath 2009), the corresponding emergent intensity of an emission line integrated along a line of sight is then simply

$$I_{ij} = \frac{h\nu_{ij}}{4\pi} \frac{n_i}{n_{tot}} A_{ij} \beta_{ij} N_{tot} = \frac{h\nu_{ij}}{4\pi} \frac{n_i}{n} A_{ij} \beta_{ij} \chi N \quad (9)$$

where χ is the abundance ratio, $\chi \equiv n_{tot}/n$, where n is the total hydrogen gas volume density (cm⁻³) and N is the hydrogen column density (cm⁻²).

Given a series of observed lines with frequency ν_{ij} , one can identify a set of characterizing parameters that best reproduces the observed line ratios and intensity magnitudes; among these parameters are the cloud's kinetic temperature (T_{kin}), velocity gradient (dv/dr), gas density n and collision partner (H and H₂) gas fractions, abundance ratio (χ), and column density (N). For a spherical geometry, this column density can then be further related to the molecular gas mass of the cloud in the following way

$$M = \pi R^2 \mu n N' \quad (10)$$

where the factor $\mu = 1.36$ takes into account the helium contribution to the molecular weight and $R = D_A \theta/2$ is the effective radius of the cloud, with D_A being the angular diameter distance to the source and θ the beam size of the line observations. N is defined in terms of the column density obtained from the LVG calculation, $N' = N(1+z)^4$ where the $(1+z)^4$ multiplicative factor reflects the decrease

in surface brightness of a source at redshift z in an expanding universe.

In the case where the emitting molecular clouds are gravitationally bound, applying the virial theorem to a homogenous spherical body yields the following constraint (Goldsmith 2001),

$$\frac{dv}{dr} \approx \sqrt{\frac{G\pi\mu mn}{15}} \approx a \sqrt{\frac{n}{(\text{cm}^{-3})}} \text{ km s}^{-1} \text{ pc}^{-1} \quad (11)$$

where $a = 7.77 \times 10^{-3}$ if $n = n_{H_2}$ and $a = 5.50 \times 10^{-3}$ if $n = n_H$. We note that the velocity gradient is inversely proportional to the dynamical time scale. Therefore, in models where the clouds are assumed to be virialized, dv/dr and n are no longer independent input parameters of the model, but rather vary according to equation (11).

For the optically thick ¹²CO J=1-0 transition line ($\beta \approx 1/\tau$), carrying out the LVG calculations with this additional virialization condition leads to a simple relation between the gas mass and the CO(1-0) line luminosity,

$$\alpha_{CO} = \frac{M_{H_2}}{L'_{CO(1-0)}} = 8.6 \frac{\sqrt{n_{H_2}/(\text{cm}^{-3})}}{T_{exc}/(\text{K})} M_{\odot} (\text{K km s}^{-1} \text{ pc}^2)^{-1} \quad (12)$$

where the excitation temperature $T_{exc} \approx T_{kin}$ when the emission line is thermalized. (In this expression we assume complete conversion to H₂ so that the gas mass is the H₂ mass.) Empirically, the Galactic value of this mass-to-luminosity ratio for virialized objects bound by gravitational forces is $\alpha = M_{H_2}/L'_{CO(1-0)} = 4.6 M_{\odot} (\text{K km s}^{-1} \text{ pc}^2)^{-1}$ (Solomon & Barrett 1991).

3 ANALYSIS OF HDF 850.1

3.1 Properties of the Galaxy

The detection of HDF 850.1, the brightest submillimetre source found in the Hubble Deep Field at a wavelength of 850 micrometers, was confirmed by Hughes et al. 1998. Since then, a full-frequency scan towards the Hubble Deep Field (beam size of 2.3'', $\Omega \approx 10^{-10}$ radians) using the IRAM (Institut de Radioastronomie Millimetrique) Plateau de Bure Interferometer and the National Radio Astronomy Observatory (NRAO) Jansky Very Large Array has detected three CO lines, identified as the CO(2-1) 230.5 GHz, CO(5-4) 576.4 GHz, and CO(6-5) 691.5 GHz rotational transitions. [CII] 1900.1 GHz (158 μm), one of the main cooling lines of the star-forming in stellar medium, has also been detected (Table 1). These lines have placed HDF 850.1 in a galaxy over density at $z = 5.183$, a redshift higher than those of

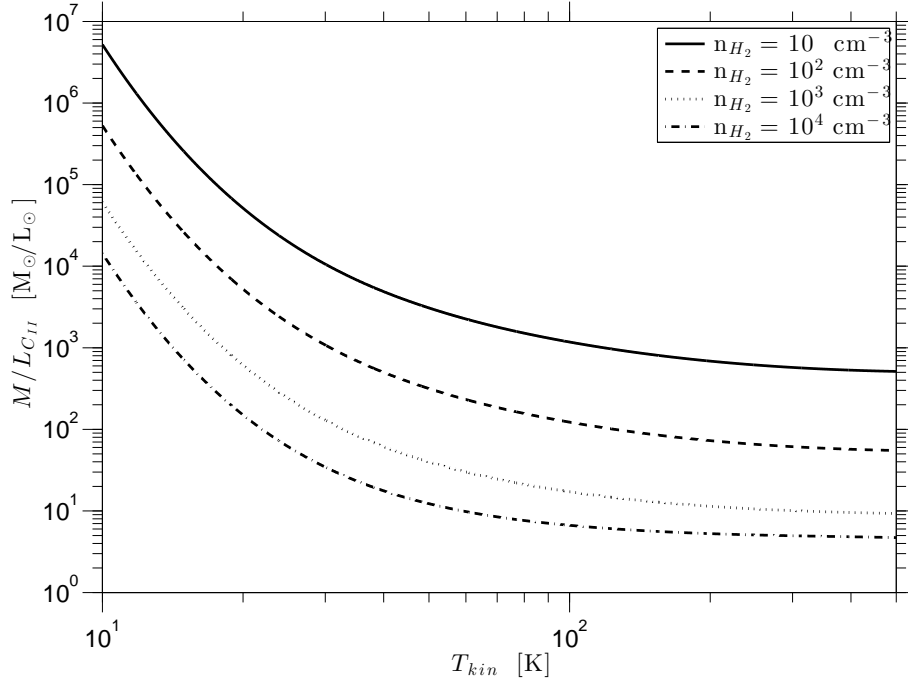


Figure 1. The gas mass to [CII] luminosity ratio for a C^+ to H_2 abundance ratio of $\chi_{C^+} = 10^{-4}$ as a function of the kinetic temperature T_{kin} at a molecular hydrogen number density of $n_{H_2} = 10, 10^2, 10^3$, and 10^4 cm^{-3} . Calculations were made in the optically thin regime, assuming the fine-structure transition $J=3/2 \rightarrow 1/2$ is due solely to spontaneous emission processes and collisions with ortho- and para- H_2 .

most of the hundreds of submillimetre-bright galaxies identified thus far (Walter et al. 2012).

Walter et al. (2012) used an LVG model to characterize the CO spectral energy distribution of HDF 850.1 and found that the observed CO line intensities could be fit with a molecular hydrogen density of $10^{3.2} \text{ cm}^{-3}$, a velocity gradient of $1.2 \text{ km s}^{-1} \text{ pc}^{-1}$, and a kinetic temperature of 45 K. Then, assuming $\alpha = 0.8 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$ as for ULIRGs, they used the 1-0 line luminosity inferred from their LVG computation to infer that $M_{H_2} = 3.5 \times 10^{10} \text{ M}_\odot$. However, as Papadopoulos et al. (2012) argue, adopting a uniform value of α for ULIRGs neglect its dependence on the density, temperature, and kinematic state of the gas; this may limit the applicability of computed conversion factors to other systems in the local or distant Universe.

Here, we broaden the LVG analysis carried out in Walter et al. 2012 and use the LVG-modeled column density to estimate the total gas mass of the source. We present several alternative models, each subject to a slightly different constraint. In particular, we first consider the case where the CO and [CII] lines originate from different regions of the molecular cloud and then consider models in which the CO molecules and C^+ ions are uniformly mixed, such that the line emissions originate in gas at the same temperature and density. In both instances, we perform our LVG computations assuming (a) gravitationally-unbound clouds and (b) virialized clouds for which the virialization condition (equation (11)) has been imposed.

To carry out these computations, we use the LVG radiative transfer code described in Davies et al. 2012. Energy levels, line frequencies and Einstein A coefficients

are taken from the Cologne Database for Molecular Spectroscopy (CDMS). The excitation and deexcitation rates of the CO rotational levels that are induced by collisions with H_2 are taken from Yang et al. (2010) while the C^+ collisional rate coefficients come from Flower & Launay (1977) and Launay & Roueff (1977).

3.2 Separate CO, C^+ Unvirialized Regions

We first consider a model in which the CO and [CII] emission lines detected at the position of HDF 850.1 originate in separate regions of the molecular gas cloud, regions which are not necessarily at the same temperature and number density. This picture is consistent with the standard structure of PDRs in which there is a layer of almost totally ionized carbon at the outer edge, intermediate regions where the carbon is atomic, and internal regions where the carbon is locked into CO (Sternberg & Dalgarno 1995; Tielens & Hollenbach 1985; Wolfire et al. 2010). In this picture then, the hydrogen is fully molecular in the CO emitting regions.

We apply our LVG model to the observed CO line intensities, without the additional virialization constraint given by equation (11). We find that the CO SED is best fit with a molecular hydrogen density of $10^{3.2} \text{ cm}^{-3}$, a kinetic temperature of 85 K, a velocity gradient $dv/dr = 5 \text{ km s}^{-1} \text{ pc}^{-1}$, and a CO to H_2 abundance ratio of $\chi_{CO} = 10^{-3.8}$. The column density that yields the correct line intensity magnitudes is $5.1 \times 10^{18} \text{ cm}^{-2}$, corresponding to a molecular hydrogen gas mass of $M_{H_2} \approx 2.54 \times 10^{10} \text{ M}_\odot$. This estimate of the gas mass is nearly 30% less than obtained by Walter et al. (2012) by applying the H_2 mass-to-CO luminosity relation

with the typically adopted conversion factor for ULIRGs, $\alpha = 0.8 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$. Given our inferred H_2 gas mass and predicted CO(1-0) line luminosity from our LVG fit, we find that $\alpha = 0.6 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$, in this model.

3.3 Separate CO, C^+ Virialized Regions

For self-gravitating clouds in virial equilibrium, the velocity gradient is no longer an independent input parameter of the LVG model, but varies with n_{H_2} according to equation (11). We calculate a three-dimensional grid of model CO line SEDs, varying T_{kin} , n_{H_2} , and χ_{CO} over a large volume of the parameter space. We find, under this virialization constraint, that the observed CO lines are fit best with a temperature of 100 K, an abundance ratio of $\chi_{\text{CO}} = 10^{-4.5}$, and a molecular hydrogen number density of $10^{2.8} \text{ cm}^{-3}$ (with a corresponding velocity gradient of $0.20 \text{ km s}^{-1} \text{ pc}^{-1}$). For this set of parameters, the beam-averaged H_2 column density is well constrained to be $N_{\text{H}_2} \approx 5.2 \times 10^{19} \text{ cm}^{-2}$, giving an associated molecular gas mass of $M_{\text{H}_2} \approx 2.59 \times 10^{11} \text{ M}_\odot$. The H_2 mass to CO luminosity conversion factor obtained in this model is $\alpha = 5.9 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$, a value similar to the Galactic conversion factor observed for virialized molecular clouds in the Milky Way, suggesting that HDF 850.1 may have some properties in common with our Galaxy. The factor-of-ten increase in the inferred mass when imposing the virialization condition is a measure of the uncertainty in the CO to H_2 mass conversion even for a standard “PDR picture”.

3.4 Optically thin [CII]

In the two models above, where the CO and [CII] lines are assumed to be emitted from separate regions of the molecular clouds, the single detected ionized carbon line is insufficient in constraining the parameters of the LVG modeled [CII] region. We thus consider the optically thin regime of the [CII] line ($\beta \simeq 1$), such that

$$I_{ij} \simeq \frac{h\nu_{ij}}{4\pi} A_{ij} x_i \chi_{\text{C}^+} N_{\text{H}_2} \rightarrow \frac{M_{\text{H}_2}}{L_{[\text{CII}]_{ij}}} = \frac{\mu m_{\text{H}_2} (1+z)^4}{h\nu_{ij} A_{ij} x_i \chi_{\text{C}^+}} \quad (13)$$

where χ_{C^+} is the C^+ to H_2 abundance ratio (fixed at a value of 10^{-4} in these calculations) and x_i represents the fraction of ionized carbon molecules in the i th energy level. Assuming that the fine-structure transition $J=3/2 \rightarrow 1/2$ is due solely to spontaneous emission processes and collisions with ortho- and para- H_2 (radiative trapping can be neglected in the optically thin regime), the equations of statistical equilibrium reduce to a simplified form and can be solved to obtain $x_{(J=3/2)}$ as a function of temperature and H_2 number density. In Fig. 1, we have plotted the resulting mass-to-luminosity ratio, $M_{\text{H}_2}/L_{[\text{CII}]}$, as a function of the kinetic temperature for several different values of n_{H_2} . Given the high [CII]/far-infrared luminosity ratio of $L_{[\text{CII}]} / L_{\text{FIR}} = (1.7 \pm 0.5) \times 10^{-3}$ in HDF 850.1 (Walter et al. 2012), it is reasonable to assume the ionized carbon is emitting efficiently and to thus consider the high temperature, large number density limit ($T_{\text{kin}} \simeq 500 \text{ K}$, $n_{\text{H}_2} \simeq 10^4 \text{ cm}^{-3}$). In this limit, the mass-to-luminosity ratio is $\approx 1.035 \text{ M}_\odot (\text{K km}^{-1} \text{ pc}^2)^{-1}$ and the corresponding molecular gas mass of the C^+ region, using the detected line intensity of the ionized carbon line

$L_{[\text{CII}]} = 1.1 \times 10^{10} \text{ L}_\odot$, is found to be $M_{\text{H}_2} \approx 5.2 \times 10^{10} \text{ M}_\odot$.

3.5 Uniformly Mixed CO, C^+ Unvirialized Region

We next consider models in which the CO molecules and C^+ ions are mixed uniformly, such that the corresponding line emissions originate in gas at the same temperature, density, and velocity gradient. These models resemble conditions found in UV-opaque cosmic-ray dominated dark cores of interstellar molecular clouds where the chemistry is driven entirely by cosmic-ray ionization. For these conditions, the density fractions n_i/n for CO and C^+ depend on a single parameter, the ratio of the cloud density to the cosmic-ray ionization rate ζ (Boger & Sternberg 2005). For Galactic conditions with $\zeta \sim 10^{-16} \text{ s}^{-1}$, the C^+ to CO ratio is generally very small. However, in objects such as HDF 850.1 where the ionization rate may be much larger due to high star formation rates, this ratio may be enhanced significantly.

The LVG fit for the observed set of line intensity ratios, $\{I_{[\text{CII}]} / I_{\text{CO}(2-1)}, I_{[\text{CII}]} / I_{\text{CO}(5-4)}, I_{[\text{CII}]} / I_{\text{CO}(6-5)}\} = \{7.08 \pm 1.67, 0.96 \pm 0.19, 1.03 \pm 0.2\} \times 10^2$ (Walter et al. 2012), yields $T_{\text{kin}} = 240 \text{ K}$, $n = 10^{3.3} \text{ cm}^{-3}$ and a C^+ to CO abundance ratio of 12. To estimate the ionization rate required to produce this C^+ /CO abundance ratio, we employed the Boger & Sternberg (2005) chemical code that captures the $\text{C}^+ - \text{C} - \text{CO}$ interconversion in a purely ionization-driven chemical medium. We solve for the corresponding gas phase metallicity and cosmic-ray ionization rate pairs (Z , ζ) that yield this relatively high C^+ to CO ratio for a cloud with temperature $T=240 \text{ K}$ and density $n_{\text{H}_2}=10^{3.3} \text{ cm}^{-3}$.

Assuming a Galactic metallicity of nearly one, (a fairly reasonable assumption given the high star formation rate observed in HDF 850.1), the cosmic-ray ionization rate required to achieve this high C^+ to CO ratio is of order $\zeta \simeq 10^{-13} \text{ s}^{-1}$ (Figure 2). Compared to the cosmic-ray ionization rate ($\sim 10^{-16} \text{ s}^{-1}$) estimated for the Milky Way, the rate found in HDF 850.1 is indeed significantly enhanced. This finding is consistent with the fact that HDF 850.1 has a star-formation rate of 850 solar masses per year, a value which is larger than the measured Galactic SFR by a factor of 10^3 (Walter et al. 2012).

Furthermore, for such a high cosmic-ray ionization rate of this magnitude, *the hydrogen is primarily atomic for the implied LVG gas density*. We find that the ratio of atomic hydrogen to molecular hydrogen in the interstellar clouds is $n_{\text{H}}/n_{\text{H}_2} \approx 3$ (Figure 2). We thus replace H_2 with H as the dominant collision partner. Given the uncertain CO-H rotational excitation rates (see Shepler et al. 2007), we assume that the rate coefficients are equal to the rates for collisions with ortho- H_2 (Flower & Pineau Des For 2010). We find that the observed CO and [CII] lines together are best fit with a temperature of 240 K, a velocity gradient of $1.5 \text{ km s}^{-1} \text{ pc}^{-1}$, an atomic hydrogen number density of $n_{\text{H}} = 10^{3.3} \text{ cm}^{-3}$, and an abundance ratio (relative to H) of 7.4×10^{-5} and 6×10^{-6} for C^+ and CO respectively. The column density that yields the correct line intensity magnitudes is $3.3 \times 10^{19} \text{ cm}^{-2}$, corresponding to an atomic hydrogen gas mass of $M_{\text{H}} \approx 8.15 \times 10^{10} \text{ M}_\odot$. The molecular hydrogen mass is then $M_{\text{H}_2} = 2(n_{\text{H}_2}/n_{\text{H}})M_{\text{H}} \approx 4.99 \times 10^{10} \text{ M}_\odot$ and the total gas mass estimate is $M_{\text{gas}} \approx 1.31 \times 10^{11} \text{ M}_\odot$. The corresponding conversion factor in this model is $\alpha = 5.3 \text{ M}_\odot (\text{K}$

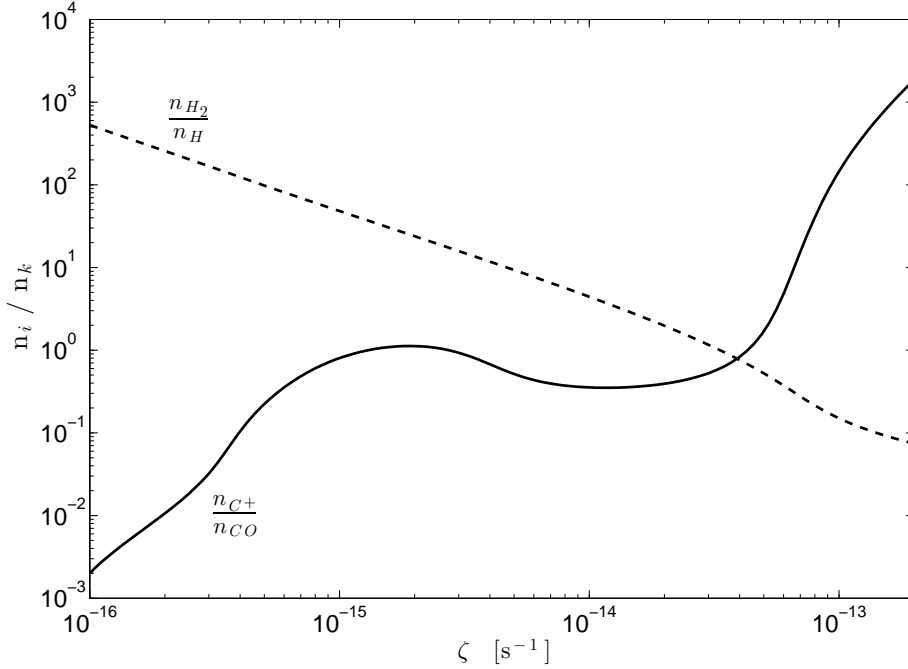


Figure 2. Dependence of the density ratios, n_{C^+}/n_{CO} (solid line) and n_{H_2}/n_H (dashed line), on the H_2 ionization rate ζ at a fixed solar metallicity $Z=1$, for our best-fit LVG parameters $T_{kin} = 240$ K and $n_{H_2} = 10^{3.3} \text{ cm}^{-3}$. As ζ increases, the abundance of C^+ relative to CO in the cosmic-ray dominated dark cores of interstellar clouds grows while that of H_2 to atomic hydrogen decreases. The desired value $n_{C^+}/n_{CO} \approx 12$ is obtained for $\zeta \approx 10^{-13} \text{ s}^{-1}$, at which point $n_H \approx 3n_{H_2}$.

$\text{km s}^{-1} \text{ pc}^2)^{-1}$, where α is now defined as the *total* gas mass to CO luminosity ratio.

3.6 Uniformly Mixed CO, C^+ Virialized Region

In the case where we assume self-gravitating clouds in virial equilibrium, we again find a best fit model with a C^+ to CO ratio of $\chi_{C^+}/\chi_{CO} \approx 10$, indicating that atomic hydrogen is the dominant collision partner in the LVG calculations. We thus calculate a three-dimensional grid of model CO and [CII] lines, varying T_{kin} , n_H , $\chi_{[CII]}$, and χ_{CO} . The observed set of CO and [CII] lines, assumed to have been emitted from the same region, are fit best with a temperature of 230 K, an atomic hydrogen number density of 10^3 cm^{-3} (with a corresponding velocity gradient of $0.17 \text{ km s}^{-1} \text{ pc}^{-1}$) and an abundance ratio of 7.3×10^{-5} and 7×10^{-6} for C^+ and CO respectively. For this set of parameters, the beam-averaged H column density is well constrained to be $N_H \approx 8.5 \times 10^{19} \text{ cm}^{-2}$. This corresponds to an atomic and molecular gas mass of $M_H \approx 2.12 \times 10^{11} M_\odot$ and $M_{H_2} \approx 1.44 \times 10^{11} M_\odot$ respectively, yielding a total gas mass estimate of $M_{gas} \approx 3.56 \times 10^{11} M_\odot$. The ratio of the total gas mass to the CO luminosity in this model is $\alpha = 8.5 M_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$.

4 COSMOLOGICAL CONSTRAINTS

Our inferred gas masses enable us to set cosmological constraints. For a particular set of cosmological parameters, the number density of dark matter halos of a given mass

can be inferred from the halo mass function. The Sheth-Tormen mass function, which is based on an ellipsoidal collapse model, expresses the comoving number density of halos n per logarithm of halo mass M as,

$$n_{ST}(M) = \frac{dn}{d \log M} = A \sqrt{\frac{2a}{\pi}} \frac{\rho_m}{M} \frac{d\nu}{d \log M} \left(1 + \frac{1}{(a\nu^2)^p}\right) e^{-a\nu^2/2} \quad (14)$$

where a reasonably good fit to simulations can be obtained by setting $A = 0.322$, $a = 0.707$, and $p = 0.3$ (Sheth & Tormen 1999). Here, ρ_m is the mean mass density of the universe and $\nu = \delta_{crit}(z)/\sigma(M)$ is the number of standard deviations away from zero that the critical collapse overdensity represents on mass scale M . Integrating this comoving number density over a halo mass range and volume element thus yields N , the total number of halos observed within solid angle A with mass greater than some M_h and redshift larger than some z ,

$$N(z, M_h) = \frac{c}{H_0} A \int_z^\infty dz \frac{D_A(z)^2}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \int_{M_h}^\infty d \log M \frac{dn}{d \log M}. \quad (15)$$

where H_0 is the Hubble constant, D_A is the angular diameter distance, and Ω_m and Ω_Λ are the present-day density parameters of matter and vacuum, respectively.

Given the detection of HDF 850.1, we can say that out of the hundreds of submillimetre-bright galaxies identified so far, at least one has been detected in the Hubble Deep Field at a redshift $z > 5$ with a halo mass greater than or equal to the halo mass associated with this source. This observation, taken together with the theoretical number density

Table 2. LVG Model: Best Fit Parameters and Results

Model Parameters	Separate, unvirialized	Separate, virialized	Mixed, unvirialized	Mixed, virialized
T_{kin} [K]	85	100	240	230
dv/dr [km s ⁻¹ pc ⁻¹]	5.0	0.2	1.5	0.17
n_{H_2} [cm ⁻³]	10 ^{3.2}	10 ^{2.8}	10 ^{2.8}	10 ^{2.5}
n_H [cm ⁻³]	–	–	10 ^{3.3}	10 ³
χ_{CO/H_2}	10 ^{-3.8}	10 ^{-4.5}	10 ^{-4.7}	10 ^{-4.7}
$\chi_{CO/H}$	–	–	10 ^{-5.3}	10 ^{-5.2}
N_{H_2} [10 ¹⁹ cm ⁻²]	0.5	5.2	1.0	2.9
N_H [10 ¹⁹ cm ⁻²]	–	–	3.3	8.5
Model Results				
M_{H_2} [M _⊙]	2.54×10 ¹⁰	2.59×10 ¹¹	4.99×10 ¹⁰	1.44×10 ¹¹
M_H [M _⊙]	–	–	8.15×10 ¹⁰	2.12×10 ¹¹
M_{gas} [M _⊙]	2.54×10 ¹⁰	2.59×10 ¹¹	1.31×10 ¹¹	3.56×10 ¹¹
$L_{CO(1-0)}$ [(K km s ⁻¹ pc ²)]	3.95×10 ¹⁰	4.41×10 ¹⁰	2.49×10 ¹⁰	4.18×10 ¹⁰
α^a [M _⊙ (K km s ⁻¹ pc ²) ⁻¹]	0.6	5.9	5.3	8.5

^a α here is defined as the ratio of total gas mass to CO(1-0) luminosity, $\alpha = M_{gas}/L'_{CO(1-0)}$. In the models where the CO and [CII] lines are assumed to be originating from separate regions, $M_{gas} = M_{H_2}$ since estimates of the atomic hydrogen gas mass could not be obtained via the LVG calculations. In the models where the CO molecules and C⁺ ions are assumed to be uniformly mixed, the total gas mass is the sum of the molecular and the atomic gas masses, $M_{gas} = M_{H_2} + M_H$.

predicted by the Sheth-Tormen mass function, constrains the halo mass associated with HDF 850.1 in the following way,

$$N(5, M_{h,min}) > 1 \quad (16)$$

where the solid angle covered by the Hubble Deep Field is $A_{HDF} = 4.4$ arcmin² (Flynn et al. 2001) and a Λ CDM cosmology is assumed with $H_0 = 70$ km s⁻¹ Mpc⁻¹, $\Omega_\Lambda = 0.7$, and $\Omega_m = 0.3$ (Komatsu et al. 2011). $M_{h,min}$, the minimum inferred halo mass for HDF 850.1, is related to the halo's minimum baryonic mass component, a quantity derived in §3 via the LVG technique, in the following way,

$$M_{h,min} = \frac{\Omega_m}{\Omega_b} M_{b,min} \quad (17)$$

where the baryonic and the total matter density parameters are $\Omega_b = 0.05$ and $\Omega_m = 0.27$ respectively. Each model's estimate of the minimum baryonic mass associated with HDF 850.1 corresponds to an estimate of N , the number of halos expected to be observed at a redshift $z \geq 5$ with mass $M_{h,min}$. Given that one such source has already been observed, an atomic physics model that yields a number $N < 1$ can be ruled out at a confidence level $P = 1 - N$; conversely, a model with a baryonic mass estimate that yields $N \geq 1$ is consistent with the cosmological constraint given by equation (15) and remains a viable model candidate.

The confidence with which models can be ruled out on this basis is plotted as a function of the minimum baryonic mass estimated by the model (solid curve in Figure 3). To check the consistency of the four LVG models considered in §3 with these results, dashed lines representing the masses derived from each model are included in the plot (upper left panel). Assuming the CO and [CII] molecules are uniformly mixed in virialized clouds results in a baryonic mass $M_{b,min} = 3.56 \times 10^{11}$ M_⊙. Such a source, with a corresponding halo mass $M_h \geq 1.9 \times 10^{12}$ M_⊙, is expected to be observed 4% of

the time; this model can thus be ruled out at the 2 σ level (dotted). The models which postulate separate virialized regions (dash-dot) and mixed unvirialized regions (solid) can be ruled out with relatively less certainty, at the 1.6 σ and 0.5 σ levels, respectively. On the other hand, modeling the CO and [CII] emission lines as originating from separate, unvirialized regions (dashed), as was done by Walter et al. (2012), results in a minimum baryonic mass which is consistent with the constraint posed by equation (15). In this case, $M_{b,min} = 2.54 \times 10^{10}$ M_⊙, and we expect to find $N > 1$ halos with a halo mass $M_h \geq 5.4 \times 10^{11}$ M_⊙ at a redshift $z \geq 5$.

These baryonic masses, obtained via the LVG method, represent conservative estimates of the total baryonic content associated with HDF 850.1. In particular, mass contributions from any ionized gas or stars residing in the galaxy are not taken into account, and in the models where a layered cloud structure is assumed, the atomic hydrogen mass component is left undetermined. We therefore consider the effects on the predicted number of observed halos, $N(M_{h,min})$, if the minimum baryonic mass derived for each model is doubled. Using these more accurate estimates of HDF 850.1's baryonic mass, we find that the two models in which the virialization condition is enforced can be ruled out at the $> 2\sigma$ level (upper right panel).

For comparison, we also consider the gas masses implied by the CO(1-0) line luminosities predicted by the two models where distinct CO and C⁺ layers are assumed (lower left panel). Adopting a CO-to-H₂ conversion factor of $\alpha = 0.8$ (K km s⁻¹ pc²)⁻¹, we find that enforcing the cosmological constraint posed by the abundance of dark matter halos does not rule out either model, even if the minimum baryonic masses are doubled to account for neglected contributions to the total gas mass (lower right panel). If a conversion factor of $\alpha = 4.6$ (K km s⁻¹ pc²)⁻¹ is used, both models can

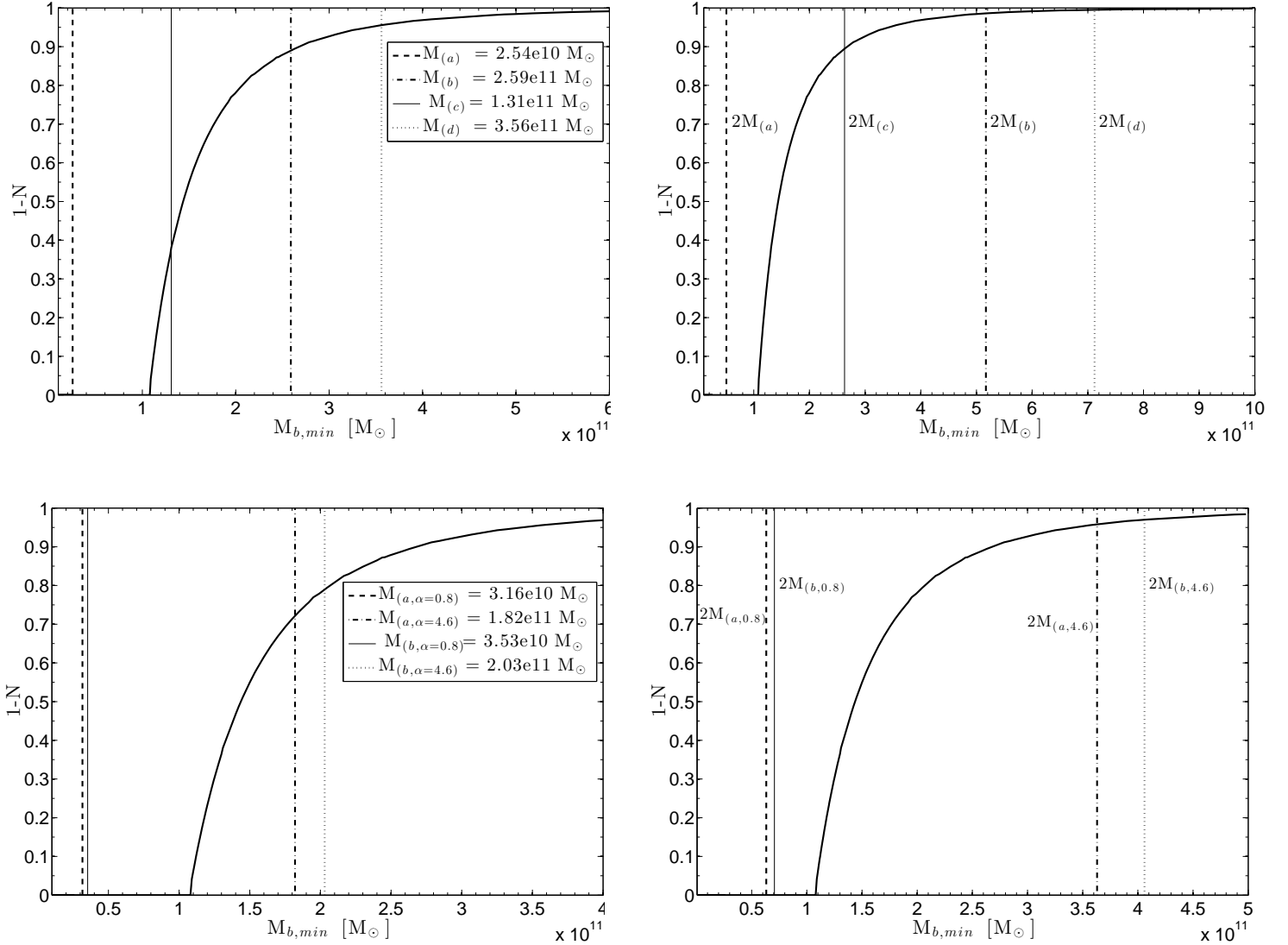


Figure 3. N represents the number of halos one expects to find with mass $M_h \geq M_{b,min}/f_b$ at a redshift $z \geq 5$ within a solid angle of $A_{HDF} = 4.4 \text{ arcmin}^2$; for $N < 1$, $1-N$ is thus the confidence with which an LVG model can be ruled out as a function of the minimum baryonic mass derived from the model (solid curve). *Upper left panel:* The vertical lines represent the mass values obtained for each of the four models presented in §3: (a) separate and unvirialized (dashed), (b) separate and virialized (dash-dot), (c) mixed and unvirialized (solid), and (d) mixed and virialized regions (dotted). In models (a) and (b), $M_{b,min} = M_{H_2}$ while in models (c) and (d), $M_{b,min} = M_{H_2} + M_H$. *Upper right panel:* The minimum baryonic masses obtained for each model were doubled to account for neglected contributions to the total gas mass; models (b) and (d) can now be ruled out at the 2.5σ and 2.8σ levels respectively. *Lower left panel:* The vertical lines represent the mass values implied by the predicted CO(1-0) line luminosities from models (a) and (b) with a CO-to- H_2 conversion factor of $\alpha = 0.8$ and $4.6 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$. If these minimum baryonic mass values are then doubled (*lower right panel*), both models can be ruled out at the $\gtrsim 2\sigma$ level in the case where $\alpha = 4.6 \text{ M}_\odot (\text{K km s}^{-1} \text{ pc}^2)^{-1}$ is adopted as the conversion factor.

be ruled out at the $\sim 1\sigma$ level and increasing these obtained masses by a factor of two drives up the sigma levels to $\gtrsim 2\sigma$ for both models.

5 SUMMARY

In this paper, we employed the LVG method to explore alternate model configurations for the CO and C^+ emission lines regions in the high-redshift source HDF 850.1. In particular, we considered emissions originating from (i) separate unvir-

alized regions, (ii) separate virialized regions, (iii) uniformly mixed unvirialized regions, and (iv) uniformly mixed virialized regions. The set of LVG parameters, $\{T_{kin}, n_{H_2}, dv/dr, \chi_{CO}, N_{H_2}\}$, were fit to reproduce the observed line ratios and the resulting column density was then used to calculate the molecular gas mass, M_{H_2} , in each case. In the models where the CO molecules and C^+ ions were assumed to be uniformly mixed, two additional parameters, n_H and N_H , were fit as well, resulting in both a molecular and an atomic hydrogen gas mass. The gas masses derived by employing

the LVG technique thus represent conservative estimates of the minimum baryonic mass associated with HDF 850.1.

These estimates were then used, together with the Sheth-Tormen mass function for dark matter halos to calculate the number of halos with mass $M_h \geq M_{b,min}$ that each model predicts to find within the HDF survey volume. Given that at least one such source has been detected, we found that models (ii), (iii), and (iv) can be ruled out at the 1.6σ , 0.5σ , and 2.0σ levels respectively. The confidence with which these models are ruled out is significantly increased if a less conservative estimate of the baryonic mass is taken; increasing the LVG-modeled gas masses by a factor of two to account for neglected contributions to the total baryonic mass, drives up these sigma levels to 2.4σ , 1.6σ , and 2.8σ respectively. We are therefore led to the conclusion that HDF 850.1 is modeled best by a collection of unvirialized molecular clouds with distinct CO and C⁺ layers, as in PDR models. The LVG calculations for this model yield a kinetic temperature of 85 K, a velocity gradient of $5.0 \text{ km s}^{-1} \text{ pc}^{-1}$, a molecular hydrogen density of $10^{3.2} \text{ cm}^{-3}$, a CO to H₂ abundance ratio of $10^{-3.8}$, and a column density of $5.1 \times 10^{18} \text{ cm}^{-2}$. The corresponding molecular gas mass obtained using this LVG approach is $M_{H_2} \approx 2.54 \times 10^{10} M_\odot$. For this preferred model we find that the CO-to-H₂ luminosity to mass ratio is $\alpha = 0.6 \text{ (K km s}^{-1} \text{ pc}^2)^{-1}$, close to the value found for ULIRGs in the local universe.

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